High-performance Penetration Depth Computation for Haptic Rendering

Young J. Kim

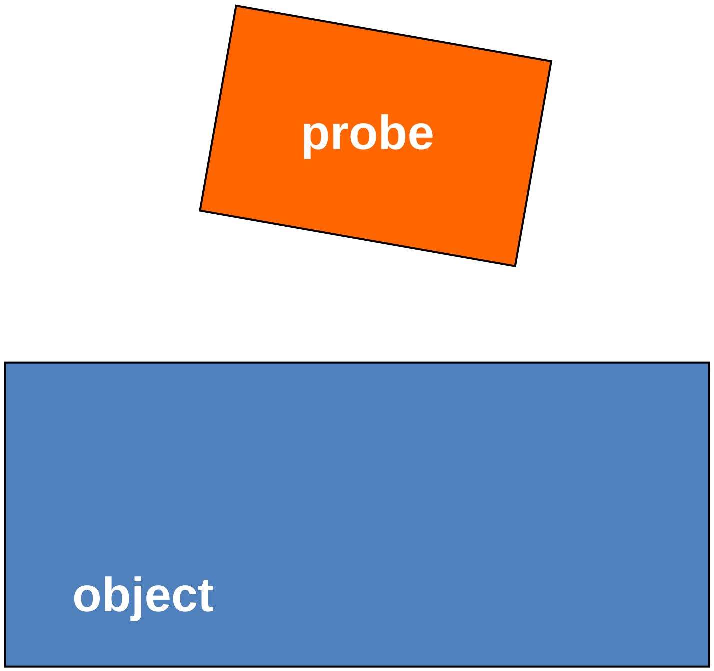
http://graphics.ewha.ac.kr

Ewha Womans University

Issues of Interpenetration

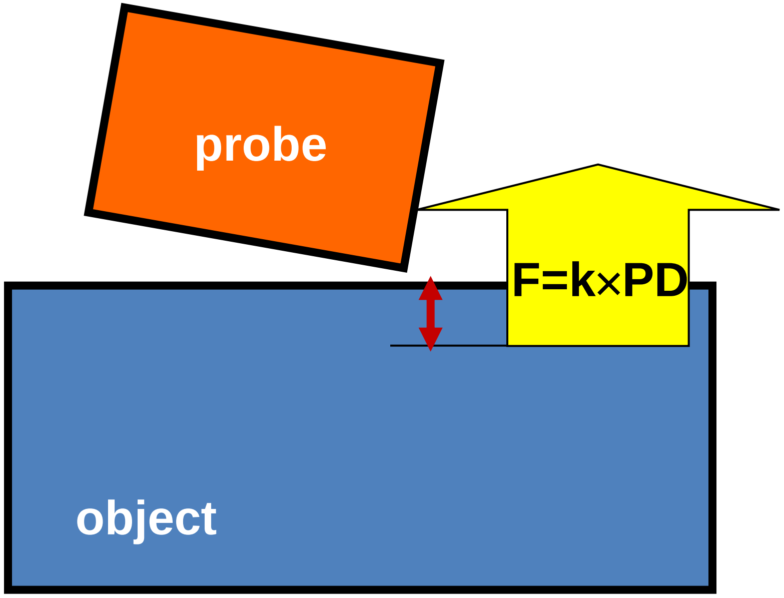
- Position and orientation of the haptic probe, governed by the user through the haptic device

- Interpenetration in haptic simulation is unavoidable



Penalty-based Response

- Penetration depth (PD) is required for computing penalty-based contact response



Previous Work on PD

- Convex polytopes -[Cameron and Culley86], [Dobkin93], [Agarwal00], [Bergen01], [Kim et al. 04]
- Non-convex polyhedra -[Kim02],[Redon and Lin06], [Lien08a,b], [Hachenberger09]
- Polygon soups – [Je et al. 12]
- Distance fields -[Fisher and Lin01], [Hoff02], [Sud06]
- Pointwise PD - [Tang et al. 09]
- Generalized PD – [Ong and Gilbert96], [Ong96], [Zhang07], [Tang et al. 12]
- Volumetric PD - [Wellner and Zachmann09]

Challenges

- Penetration depth (PD)
  - Is very expensive to compute accurately
  - May not handle arbitrary geometry and topology
- Current practice
  - Hacks

  →Slow, inconsistent, geometrically unstable

Goal

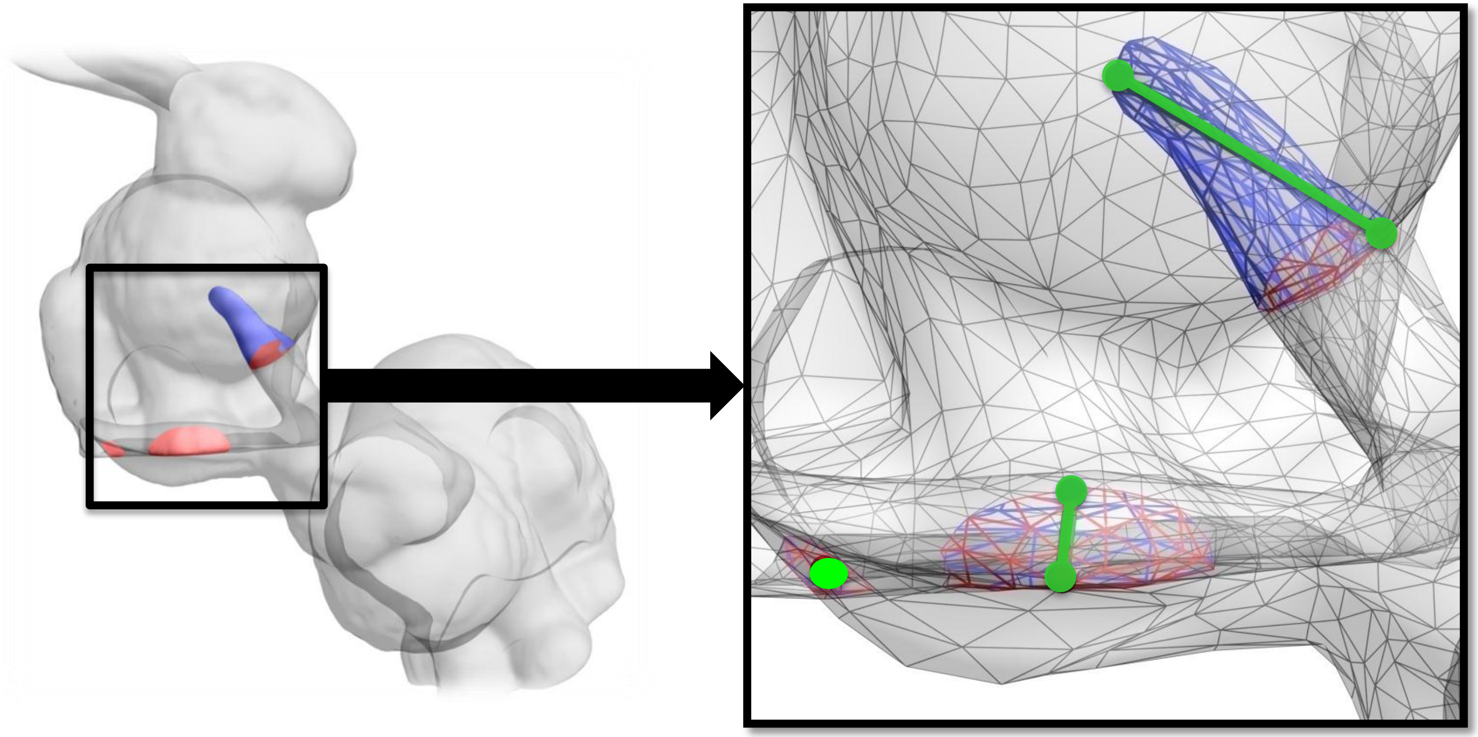
- Recent research results
  - Pointwise
  - Translational
  - Generalized
- Recent results on 6DoF haptic rendering

M. Tang, M. Lee, Y. J. Kim, Interactive Hausdorff Distance Computation for General Polygonal Models, SIGGRAPH 2009

Pointwise Penetration Depth

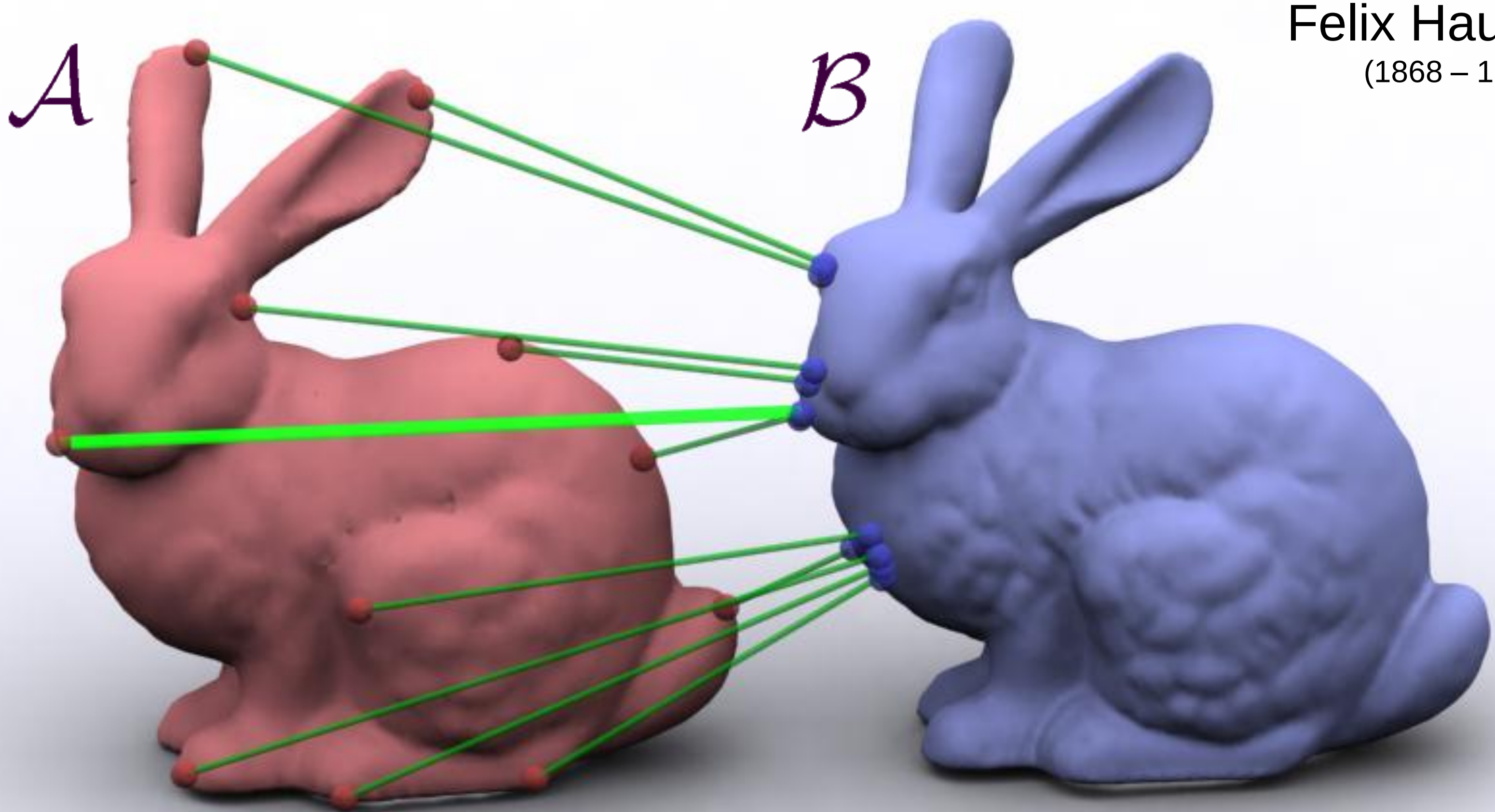
Pointwise Penetration Depth

- Defined as deepest interpenetrating points



One-sided Hausdorff Distance

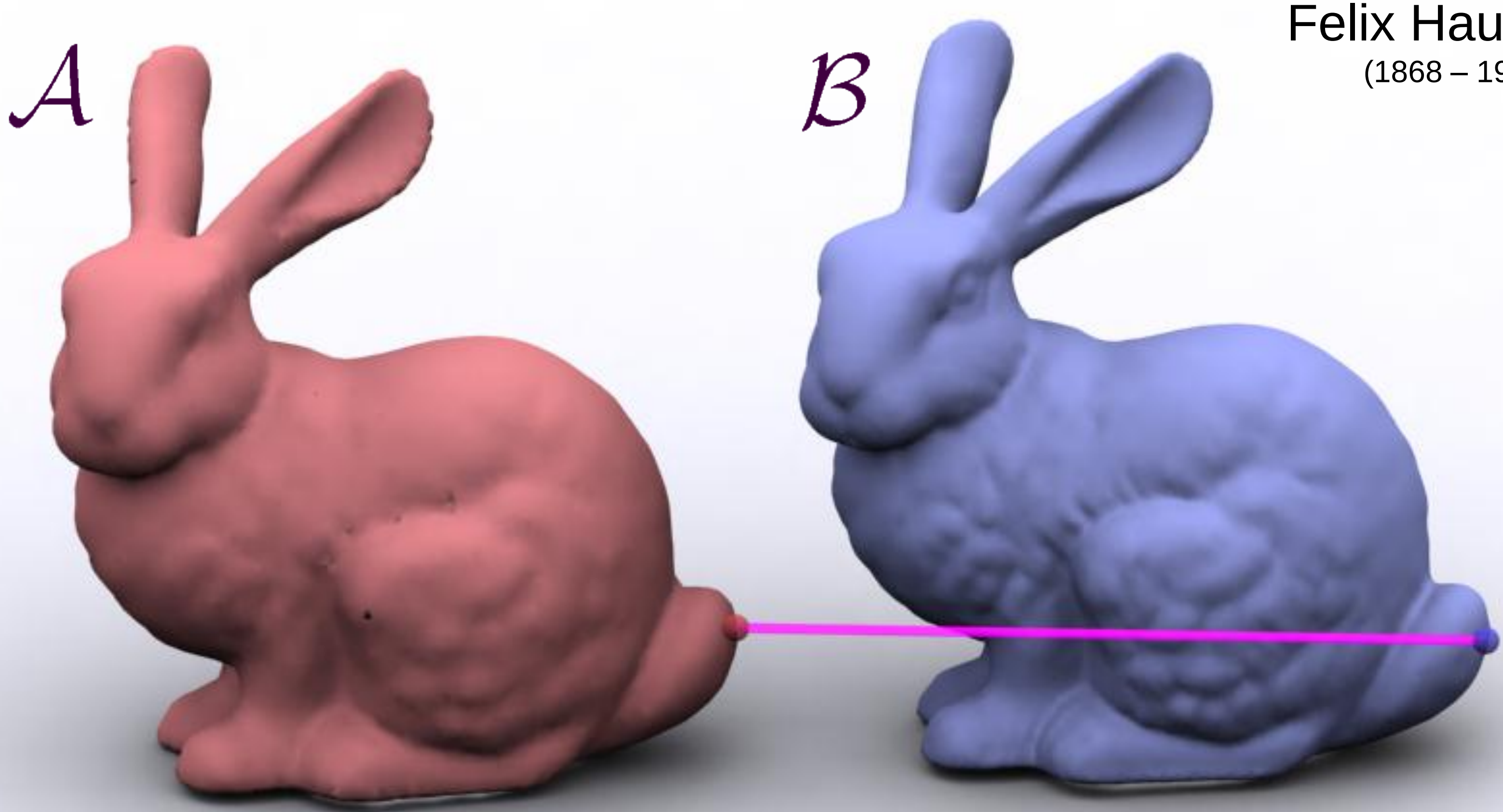
Felix Hausdorff
(1868 – 1942)



$$h(\mathcal{A},\mathcal{B}) = \max_{\mathbf{a}\in\mathcal{A}}\left(\min_{\mathbf{b}\in\mathcal{B}}\left\|\mathbf{a}-\mathbf{b}\right\|\right)$$

One-sided Hausdorff Distance

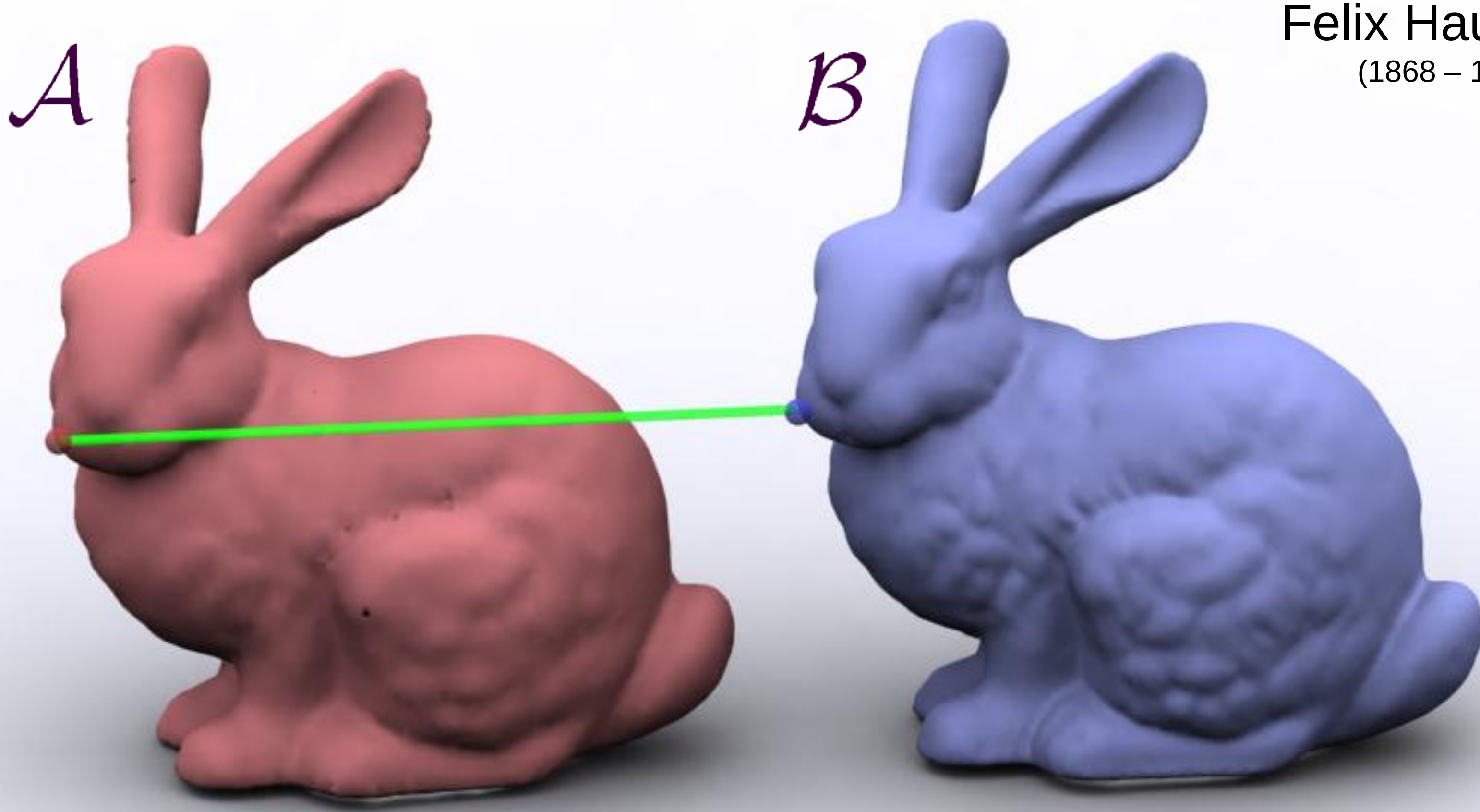
Felix Hausdorff
(1868 – 1942)



$$h(\mathcal{B},\mathcal{A}) \equiv \max_{\mathbf{b}\in\mathcal{B}}\left(\min_{\mathbf{a}\in\mathcal{A}}\|\mathbf{a}-\mathbf{b}\|\right)$$

Two-sided Hausdorff Distance

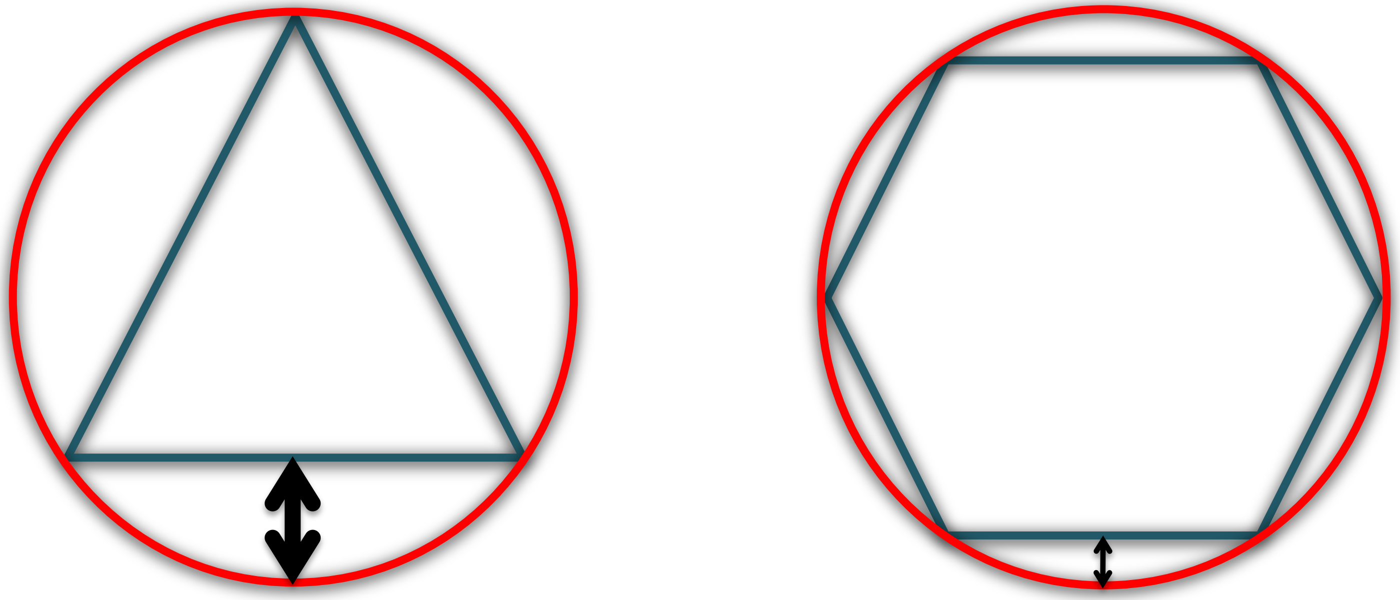
Felix Hausdorff
(1868 – 1942)



$$H(\mathcal{A},\mathcal{B}) \equiv \max\big(h(\mathcal{A},\mathcal{B}), h(\mathcal{B},\mathcal{A})\big)$$

Shape Deviation Measure

- Hausdorff distance quantifies deviation between two geometric models

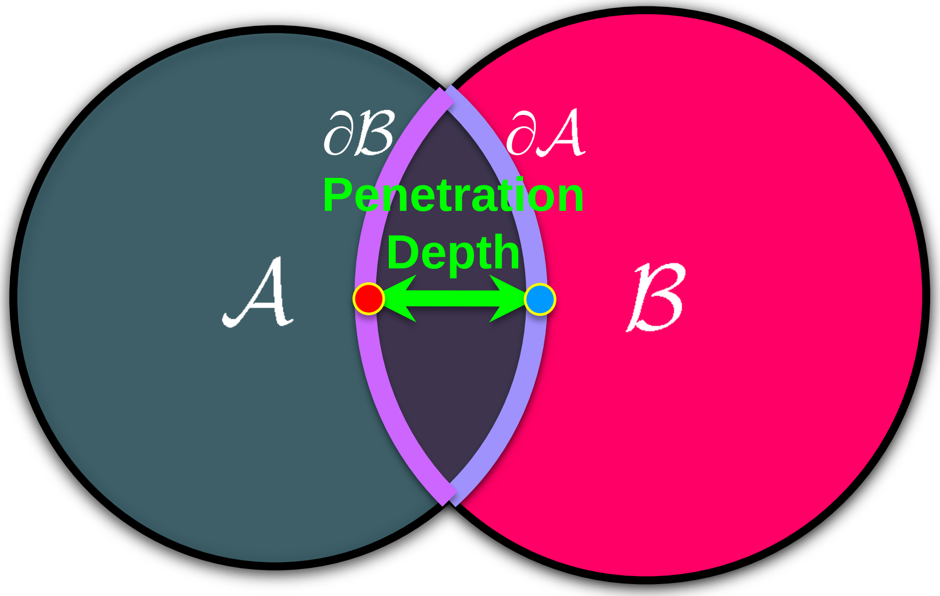


Large Hausdorff Distance Value

Small Hausdorff Distance Value

Pointwise Penetration Depth

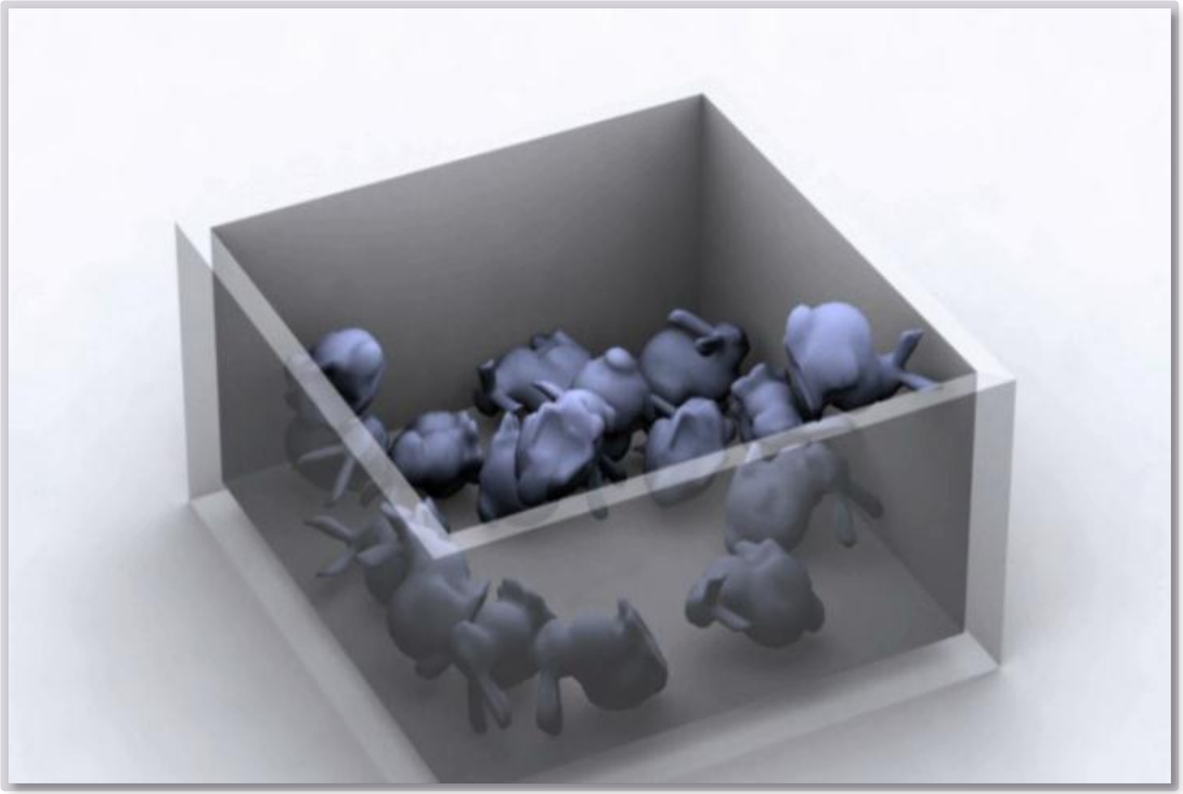
1. Find intersection surfaces $\partial\mathcal{A}$ and $\partial\mathcal{B}$
2. Penetration depth = $H(\partial\mathcal{A}, \partial\mathcal{B})$



Pointwise Penetration Depth

Demo (40K Bunny vs 40K Bunny)

Benchmark: Pointwise PD



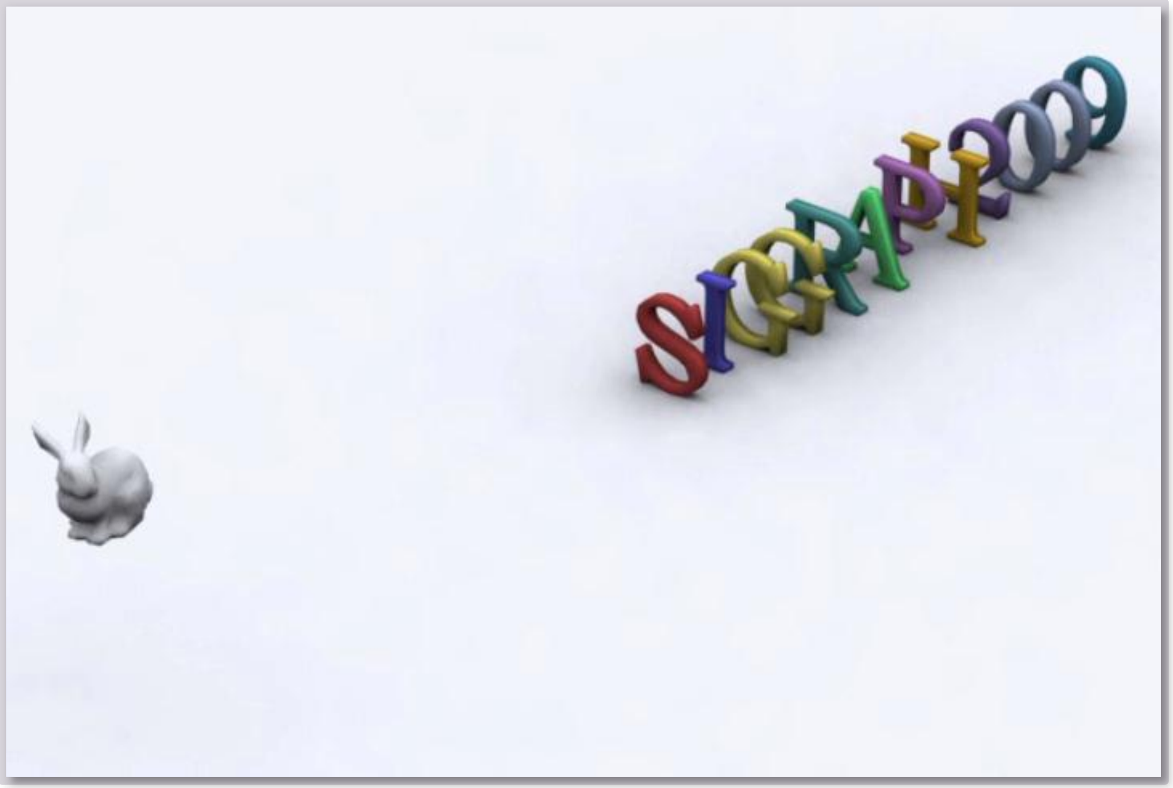
Model complexity

– 50K tri

Avg. Performance

– 3.88ms/pair

Benchmark: Pointwise PD



Model complexity
- 3.5K tri

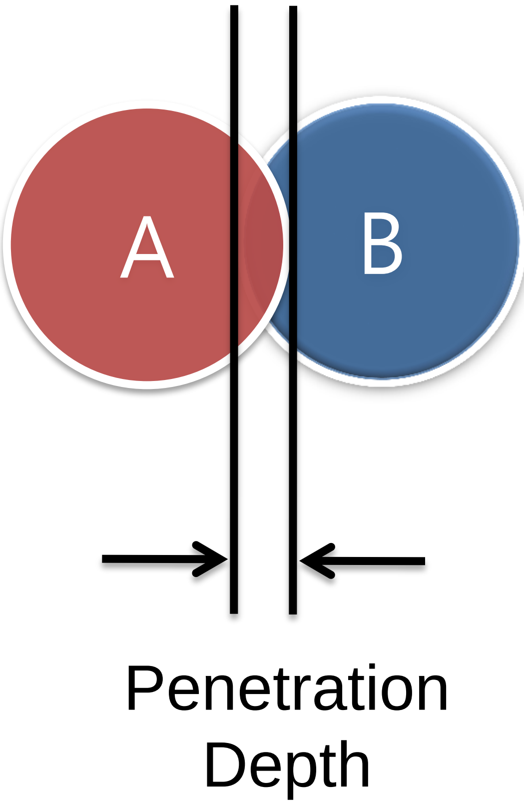
Avg. performance
- 0.95ms/pair

C. Je, M. Tang, Y. Lee, M. Lee, Y. J. Kim, PolyDepth: Real-time Penetration Depth Computation using Iterative Contact-space Projection, ACM Transactions on Graphics 2012

Translational Penetration Depth

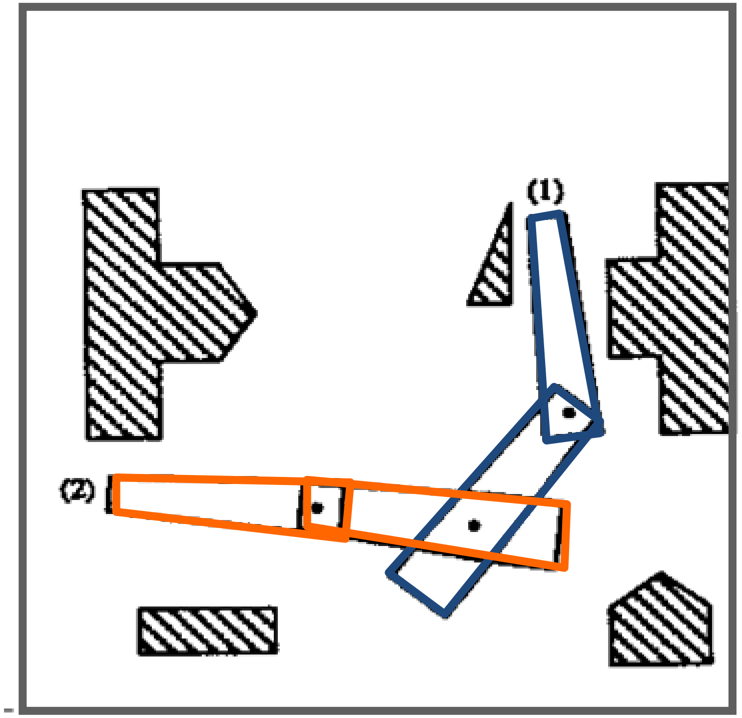
(Translational) Penetration Depth
[Dobkin 93]

- Minimum translational distance to separate overlapping objects

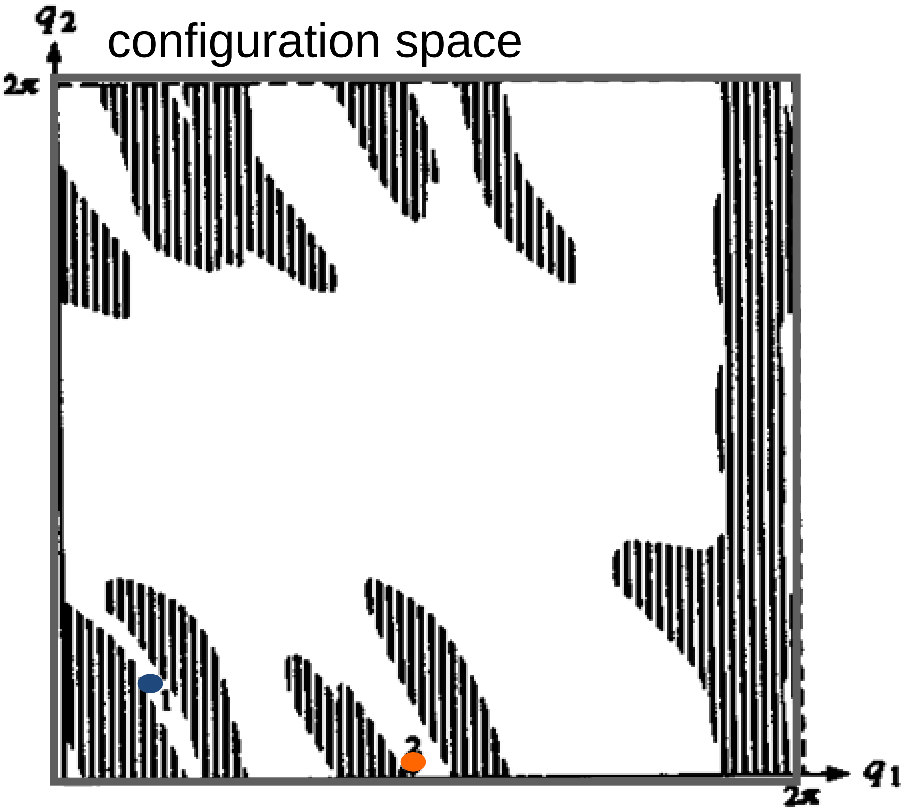


Configuration Space

workspace



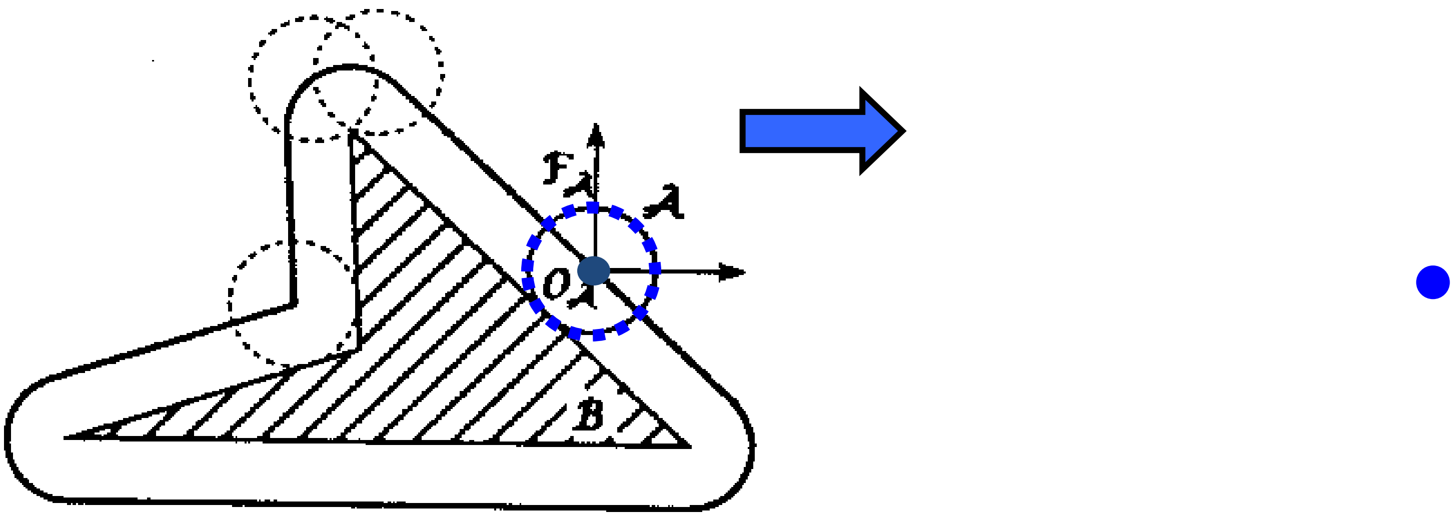
configuration space



Translational Configuration Space

workspace

configuration space

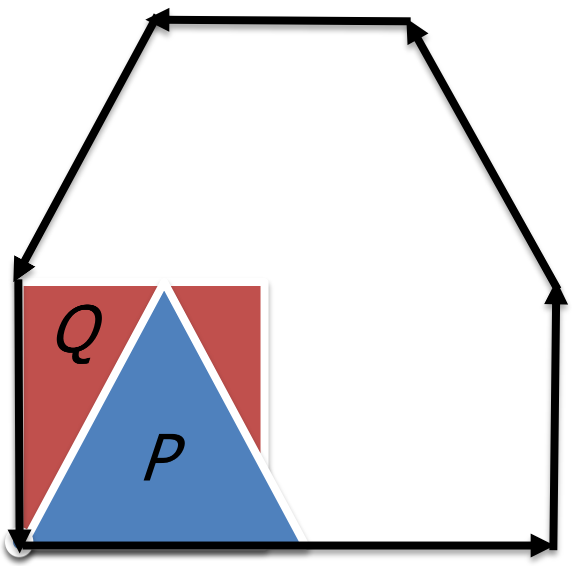


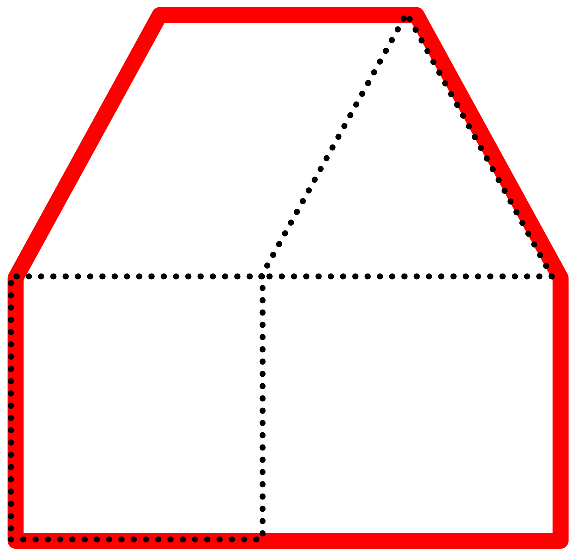
Translational C-space = Minkowski Sums

Minkowski Sum

$$P \oplus Q = \{\mathbf{p} + \mathbf{q} \mid \mathbf{p} \in P, \mathbf{q} \in Q\}$$

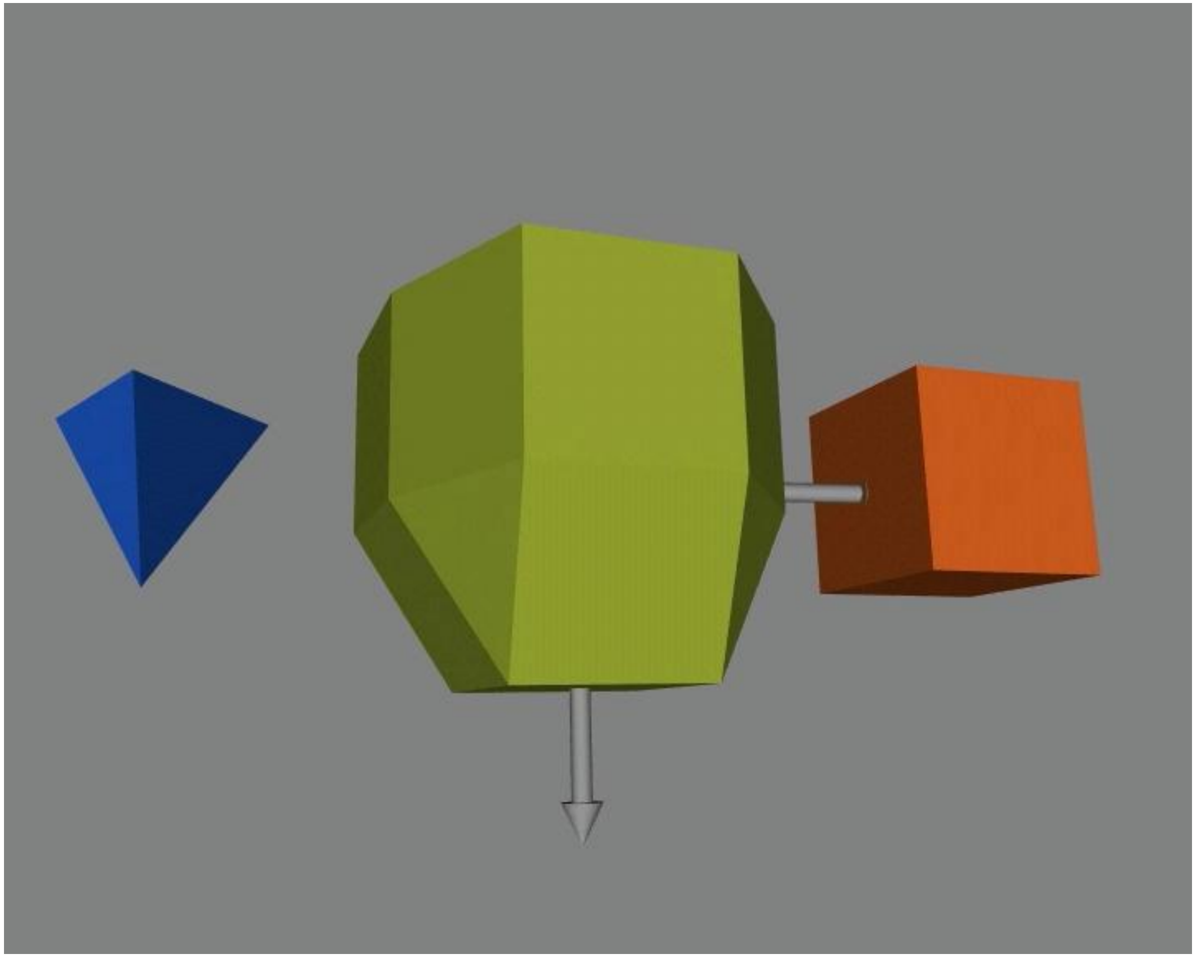
$$P \oplus -Q = \{\mathbf{p} - \mathbf{q} \mid \mathbf{p} \in P, \mathbf{q} \in Q\}$$





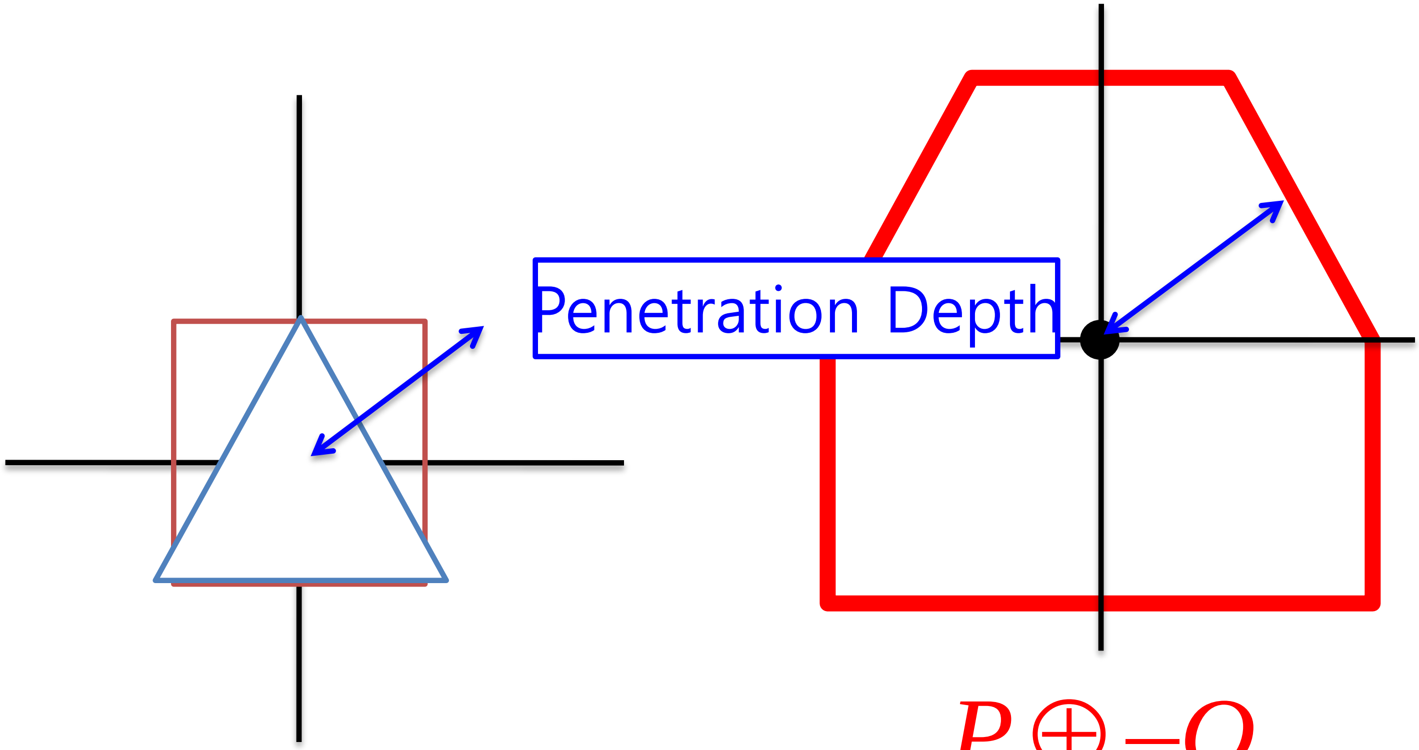
$P \oplus -Q$

Example



Video credit: D. Halperin

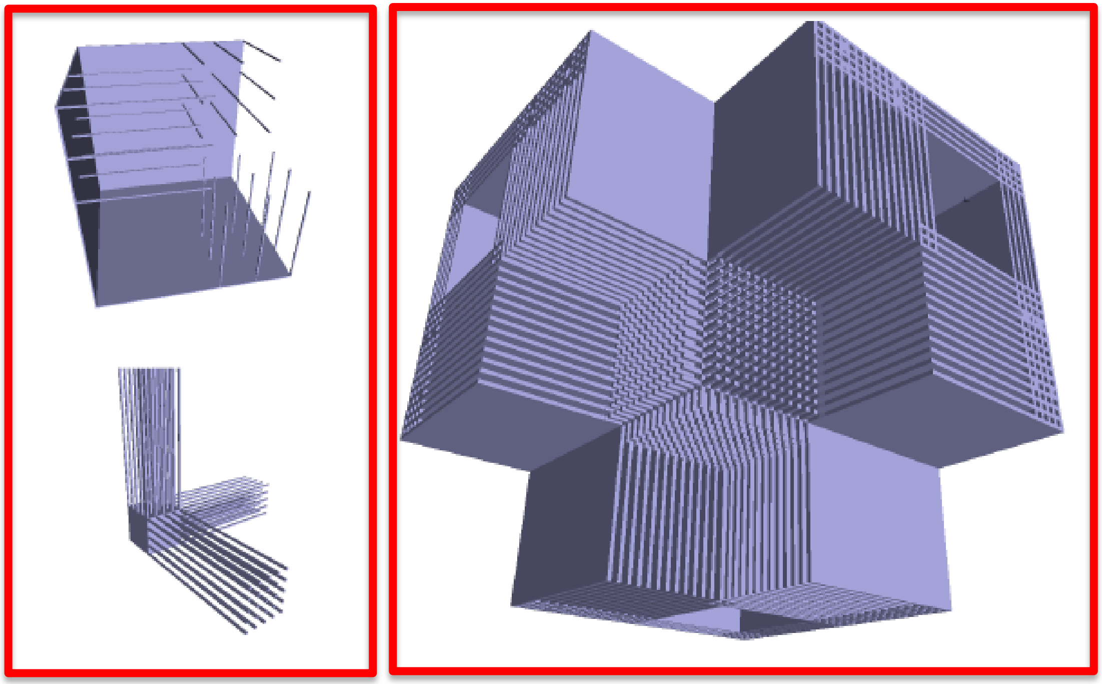
PD VS Minkowski Sum



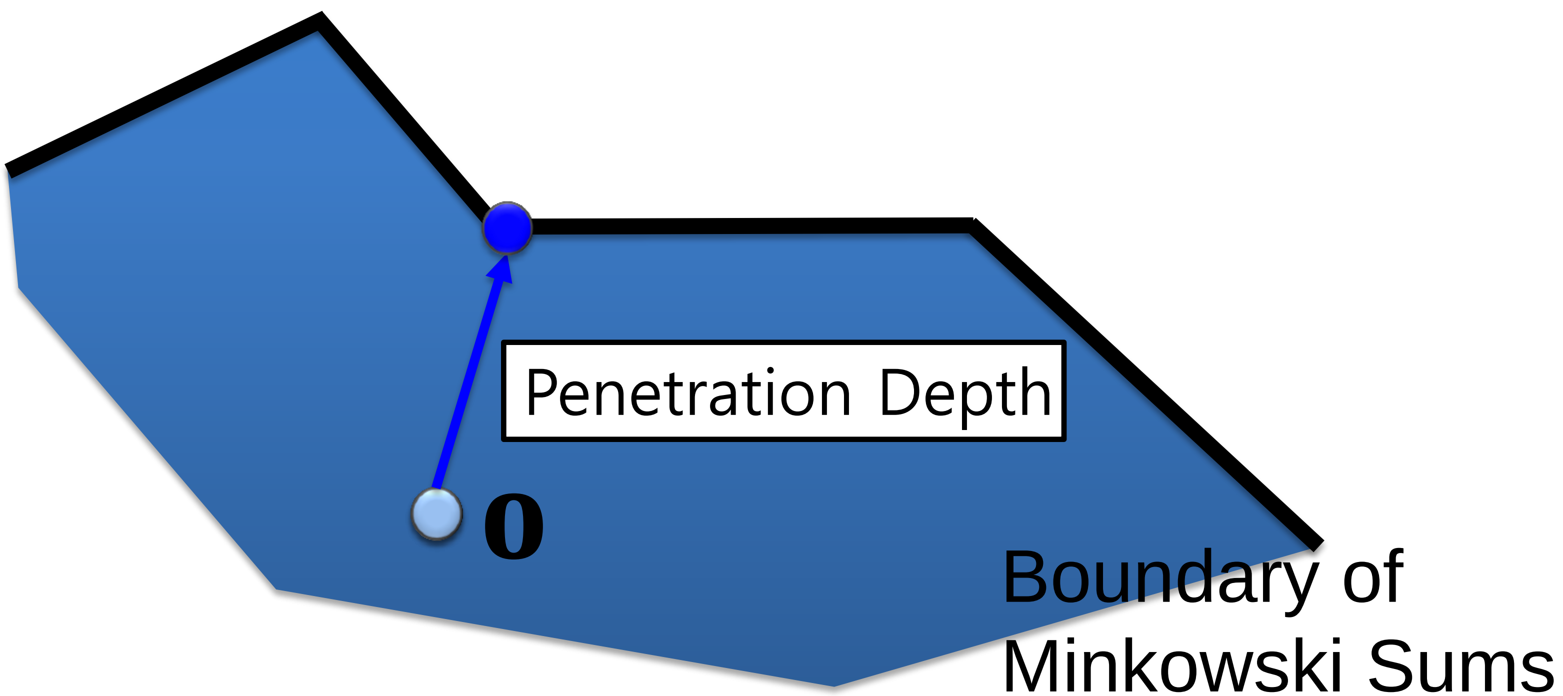
$$P \oplus -Q$$

Combinatorial Explosion

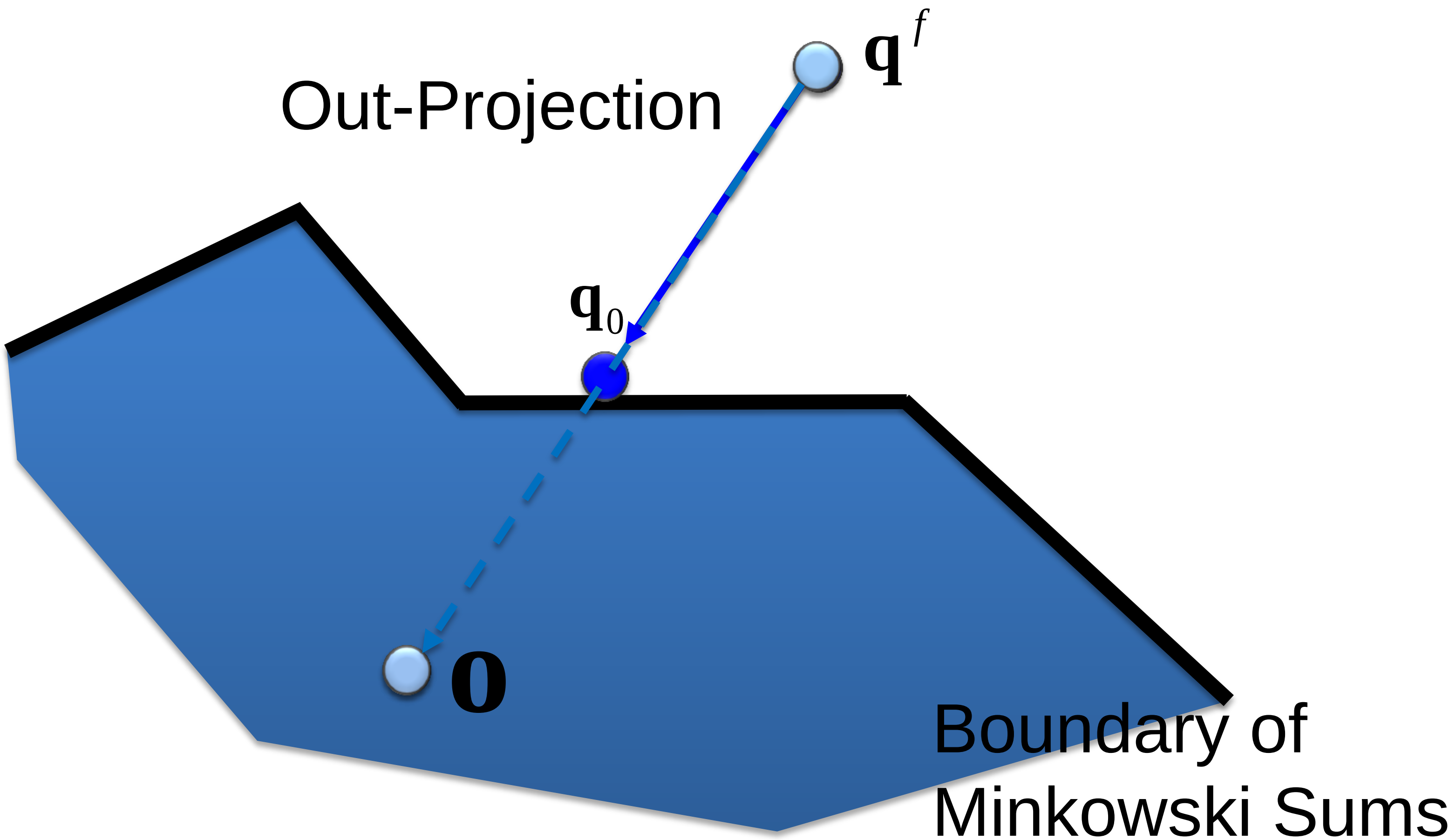
- Complexity of Minkowski Sum
  - $O(m^3n^3)$ with m and n triangles



PD Estimation



Out-Projection = Continuous Collision Detection

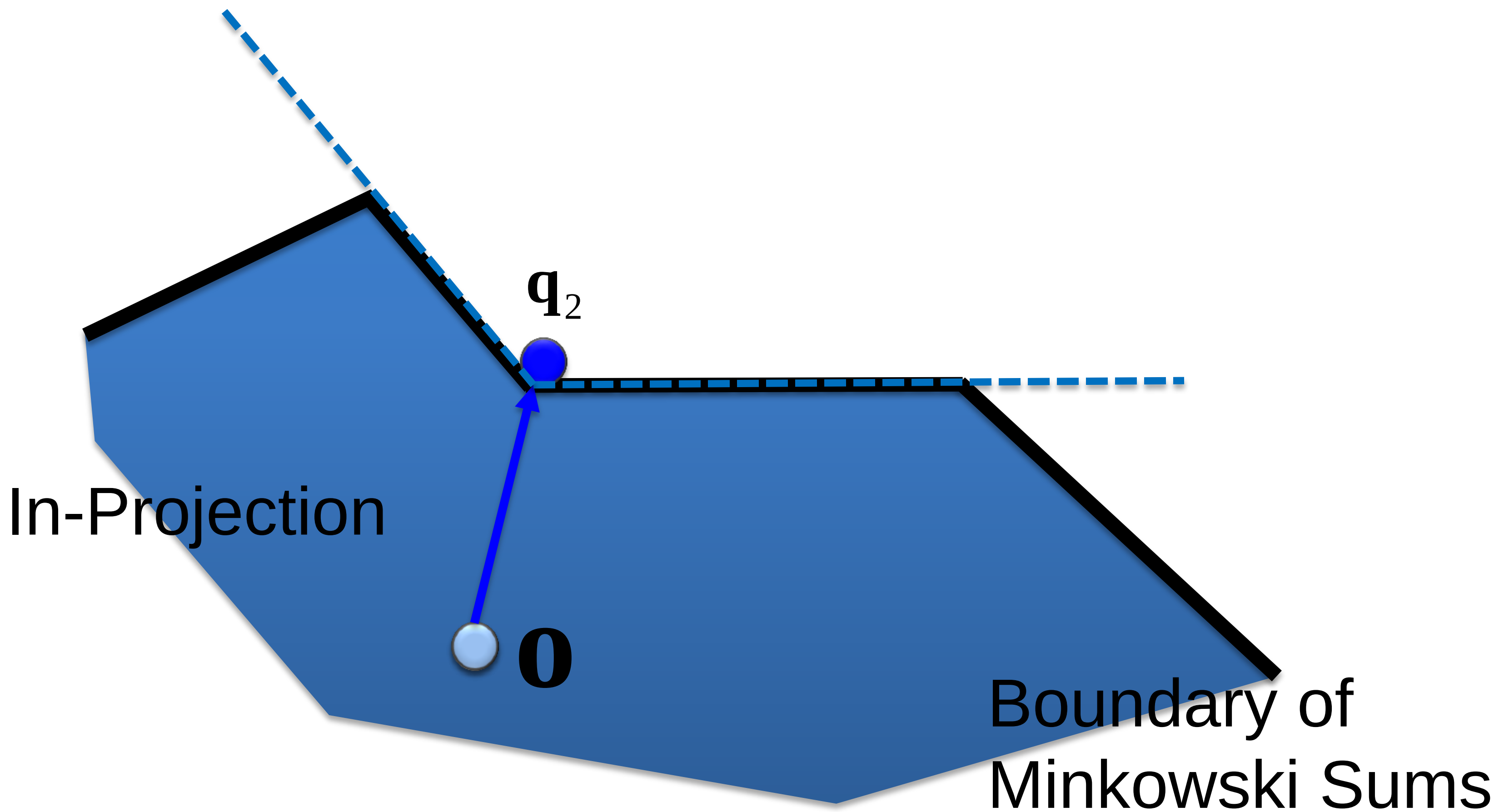


Continuous Collision Detection

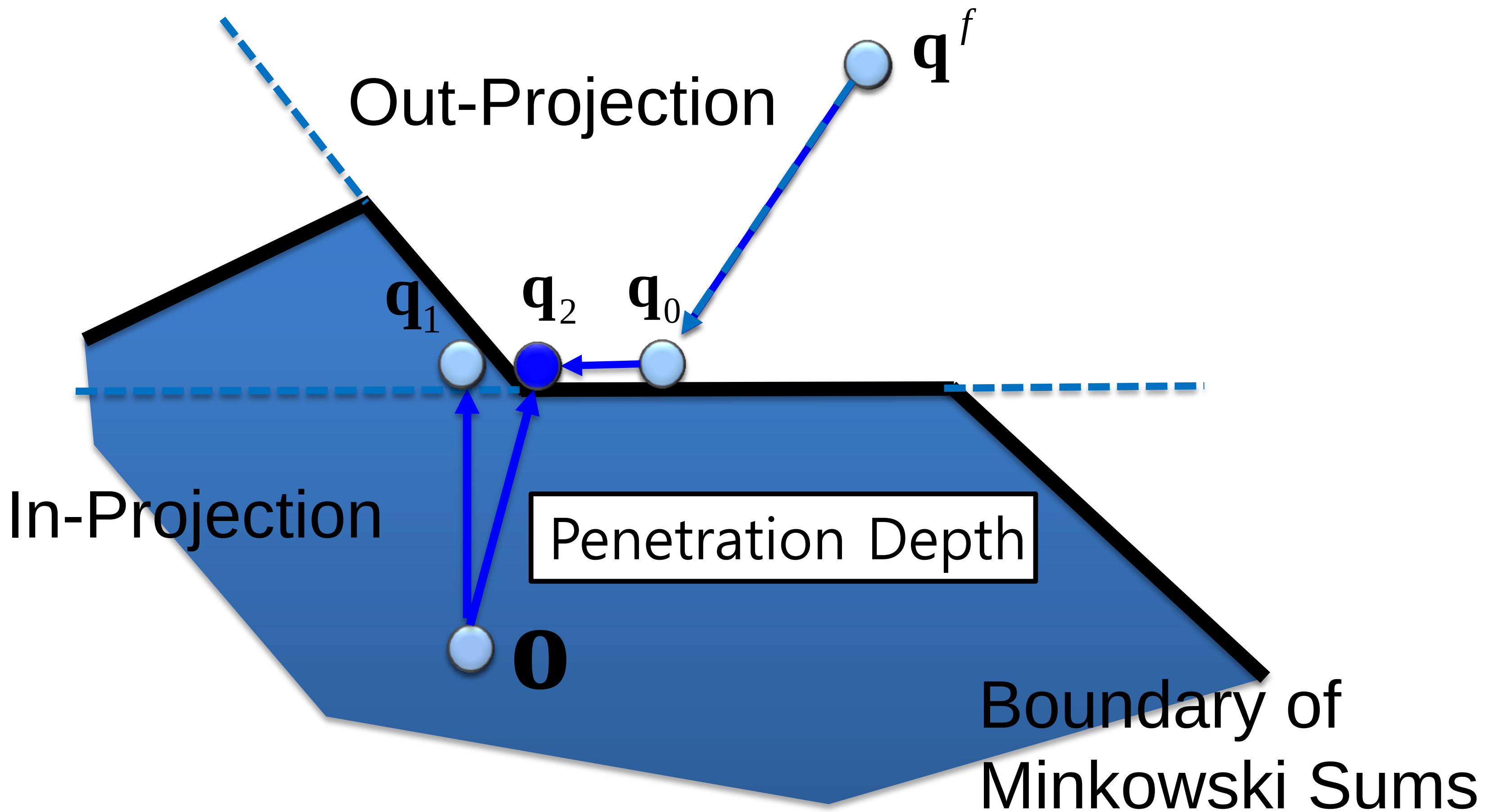
- Source codes are available
  - http://graphics.ewha.ac.kr/FAST (2-manifold)
  - http://graphics.ewha.ac.kr/C2A (polygon-soups)
  - http://graphics.ewha.ac.kr/CATCH (articulated)
  - http://graphics.ewha.ac.kr/CCQ (for motion planning)

In-Projection = LCP

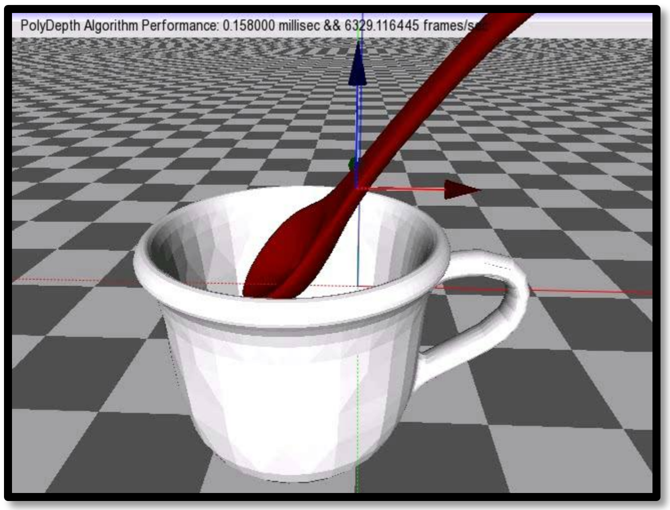
(Linear Complementarity Problem)



PolyDepth: Iterative Optimization



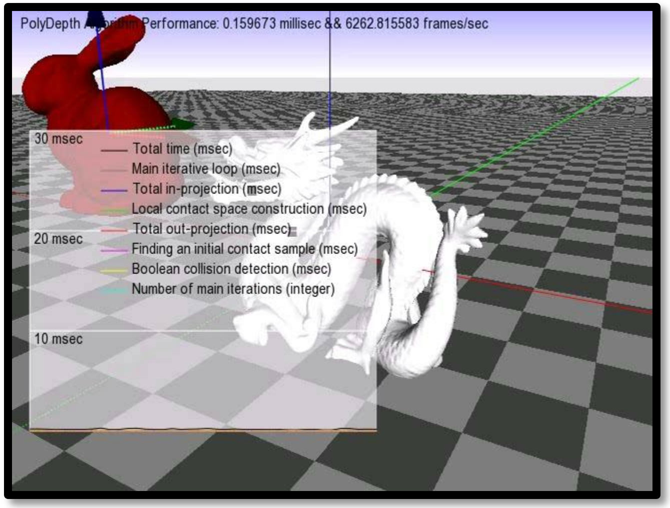
PolyDepth Performance



- Spoon: 1.3K triangles
- Cup: 8.4K triangles

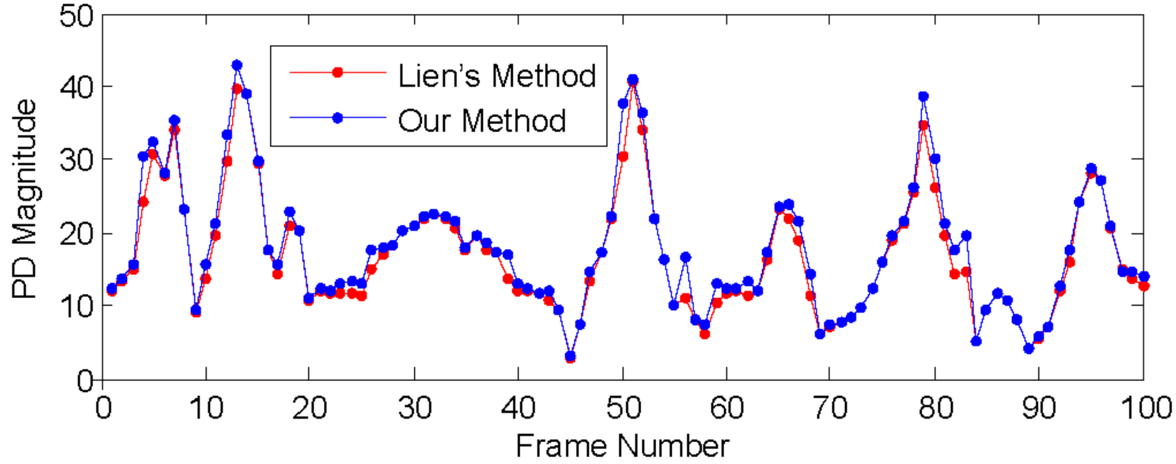
- Time: 1~7 msec

PolyDepth Performance

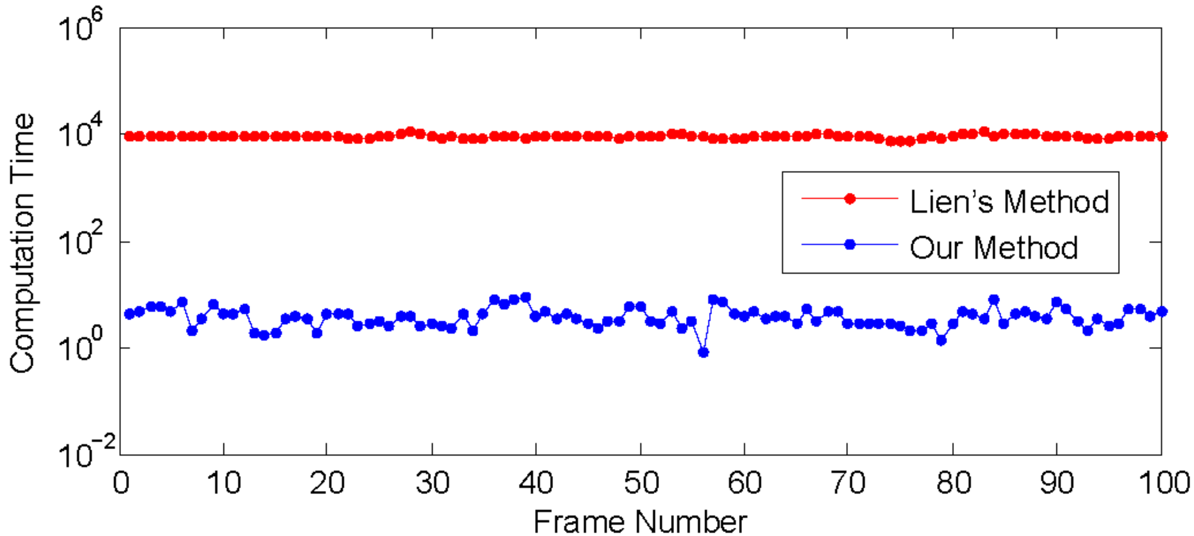


- Bunny: 40K triangles
- Dragon: 174K triangles
- Time: 2~15 msec

Comparison against Exact Solution



Accuracy



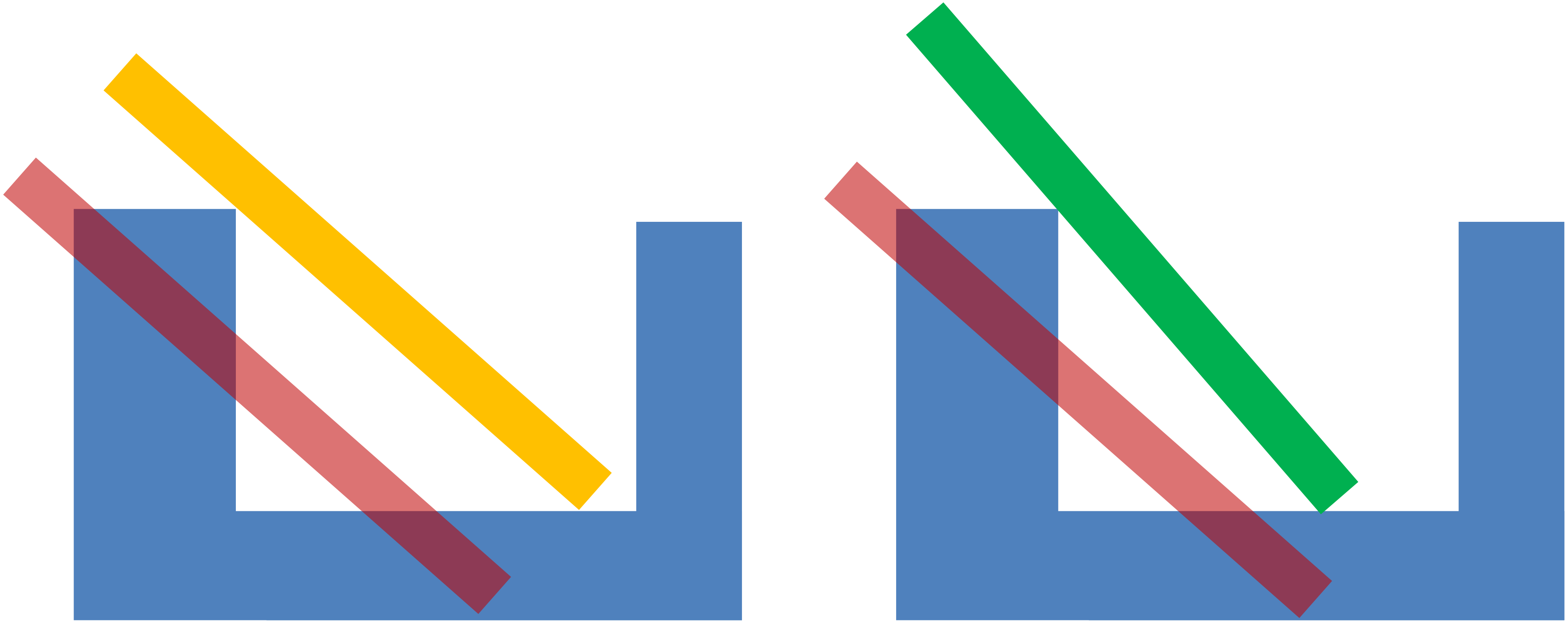
Performance

M. Tang, Y. J. Kim, Interactive Generalized Penetration Depth Computation for Rigid and Articulated Models using Object Norm, Ewha Technical Report 2012

Generalized Penetration Depth

Generalized Penetration Depth

- Minimal rigid motion to separate overlapping objects



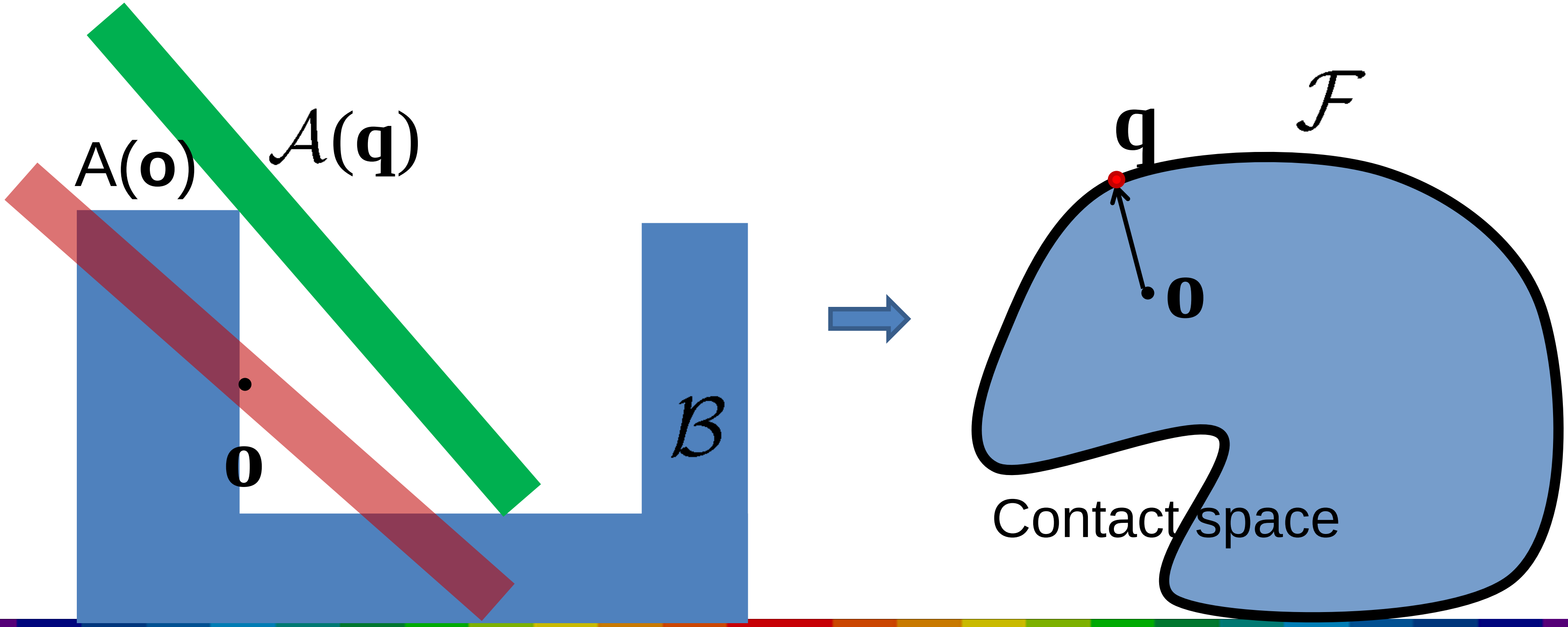
Translational PD

Generalized PD

Definition of Generalized PD

- Defined in 6D configuration space

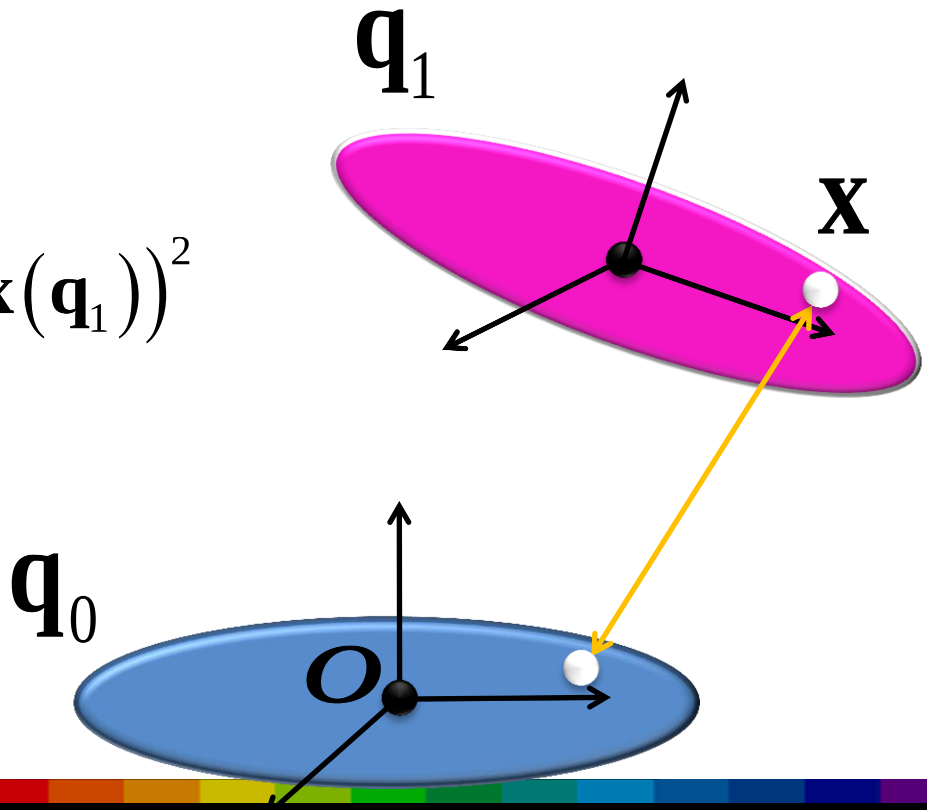
$$PD_g^{\sigma}(\mathcal{A},\mathcal{B}) = \left\{ \min\left\{ \sigma_{\mathcal{A}}(\mathbf{q},\mathbf{o}) \right\} \middle\| interior\left(\mathcal{A}(\mathbf{q})\right) \cap \mathcal{B} = \varnothing, \mathbf{q} \in \mathcal{F} \right\}$$



Distance metric

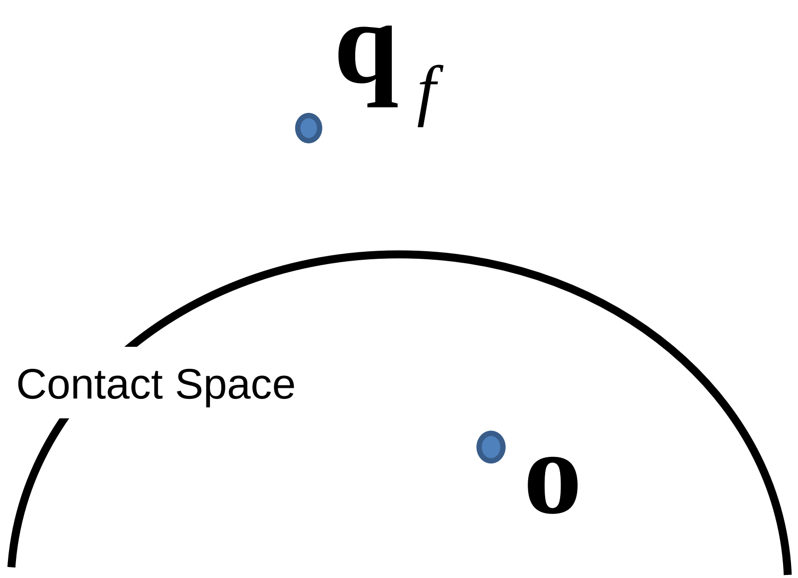
- Object norm
  - The average squared displacement

$$\sigma_{\mathcal{A}}\left(\mathbf{q}_0,\mathbf{q}_1\right)=\frac{1}{V}\int_{\mathbf{x}\in\mathcal{A}}\left(\mathbf{x}\left(\mathbf{q}_0\right)-\mathbf{x}\left(\mathbf{q}_1\right)\right)^2$$



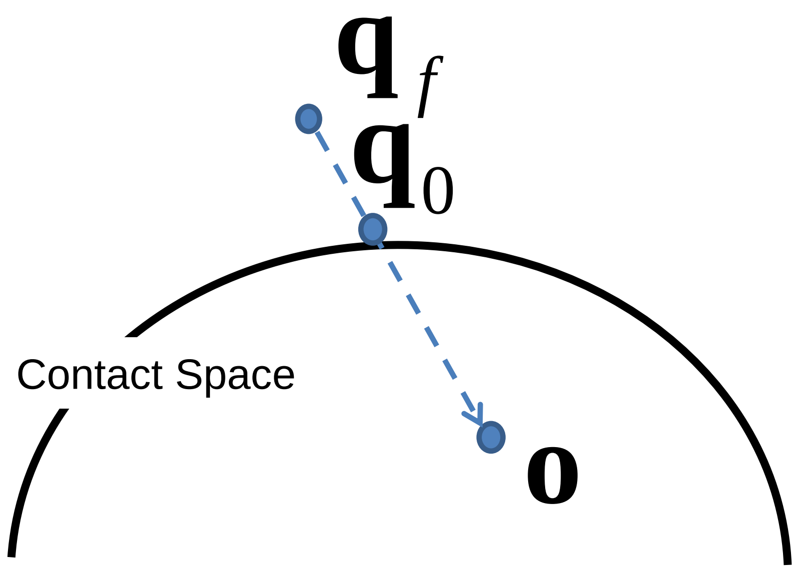
PolyDepth++ Algorithm

1. Free-configuration selection



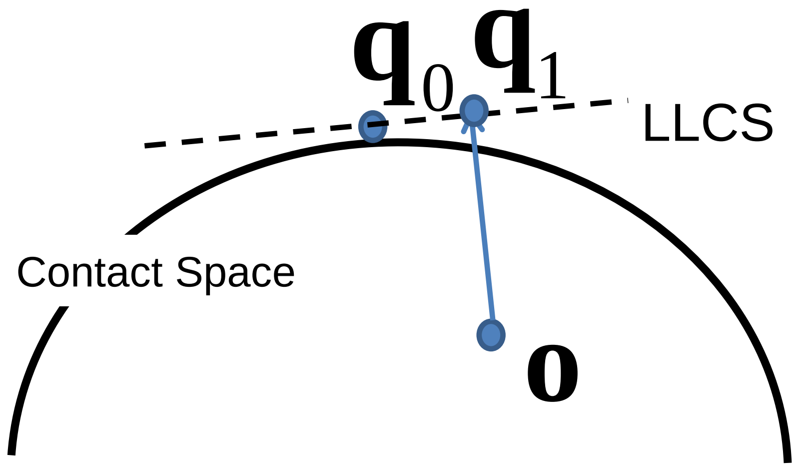
PolyDepth++ Algorithm

2. Contact-space projection



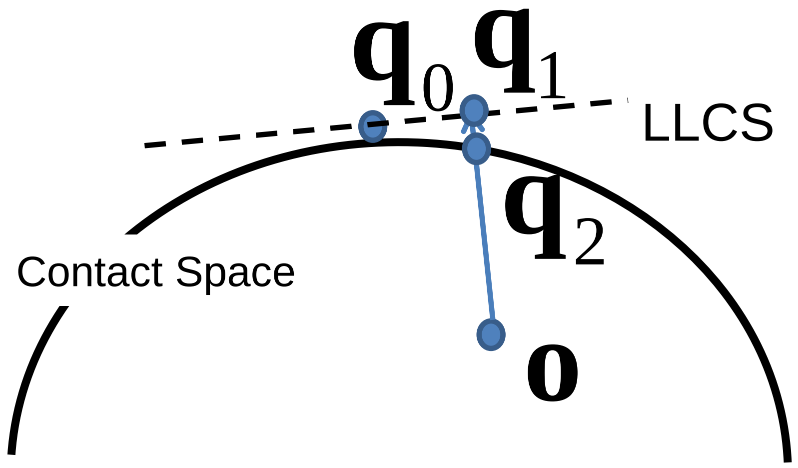
PolyDepth++ Algorithm

3. Constrained optimization



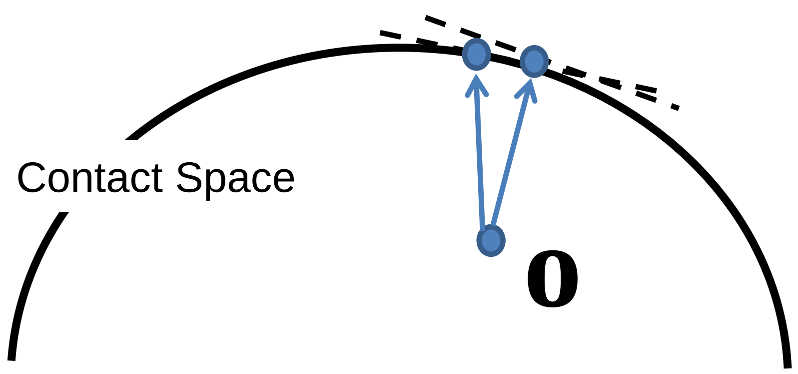
PolyDepth++ Algorithm

4. Re-projection



PolyDepth++ Algorithm

5. Iteration until finding a locally-optimal solution

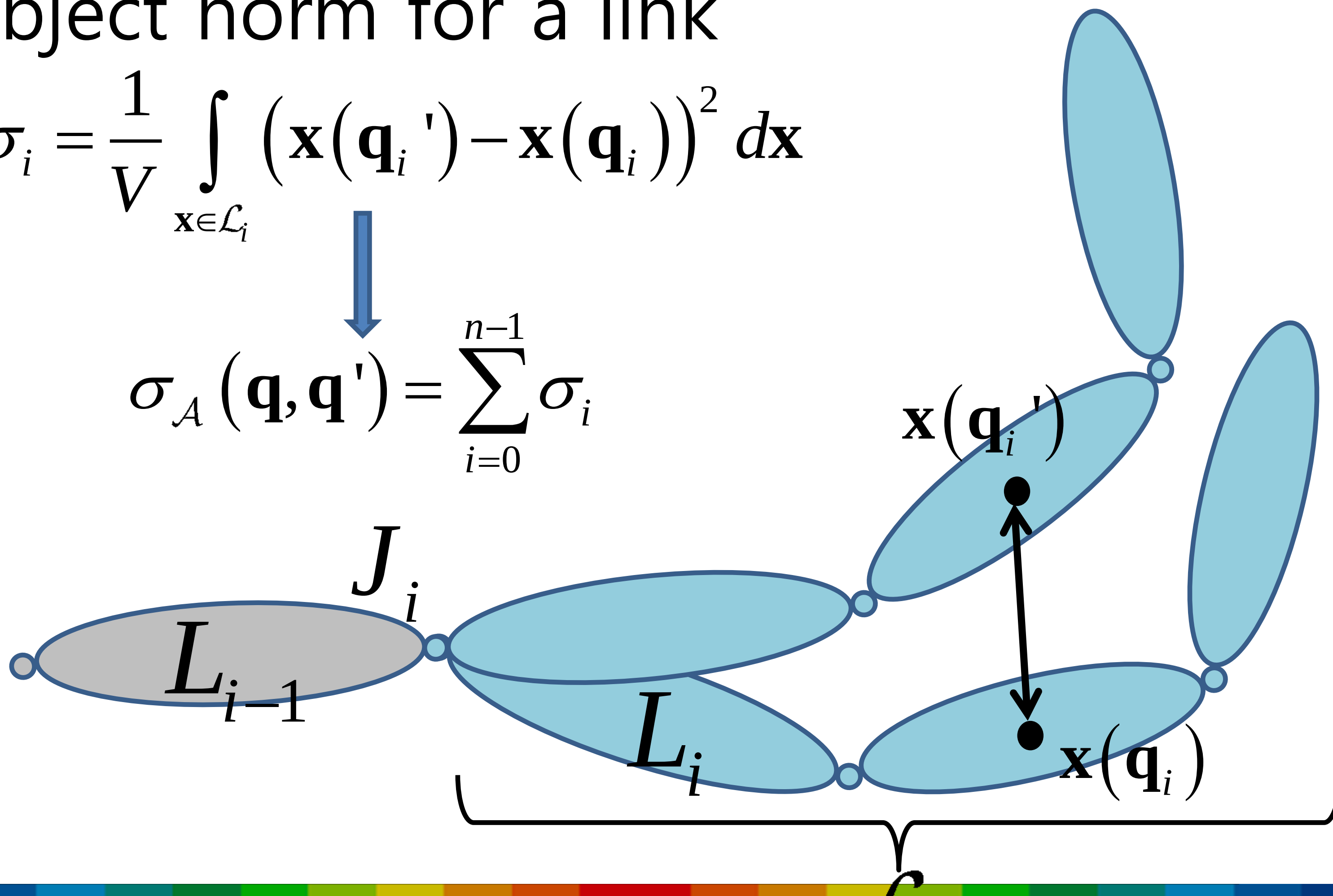


PolyDepth++ for Articulated Model

- Object norm for a link

$$\sigma_i = \frac{1}{V} \int_{\mathbf{x}\in\mathcal{L}_i} \left(\mathbf{x}(\mathbf{q}_i') - \mathbf{x}(\mathbf{q}_i)\right)^2 d\mathbf{x}$$

$$\sigma_{\mathcal{A}}(\mathbf{q},\mathbf{q}') = \sum_{i=0}^{n-1} \sigma_i$$



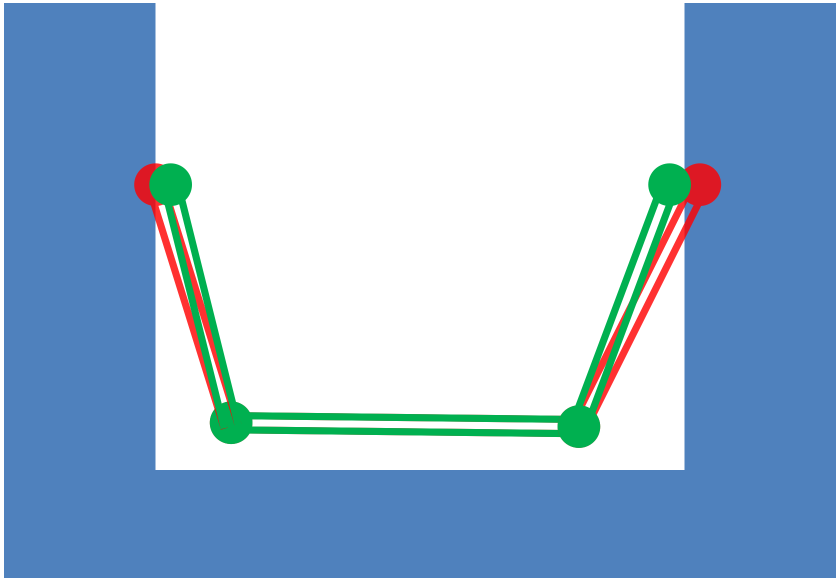
PolyDepth++ for Articulated Model

- Constrained optimization in higher dimension

$$\text{Minimize } \sigma(\mathbf{q}) = \sum_{i=0}^{n-1} \sigma_i$$

$$\text{subject to: } \mathbf{C}(\mathbf{q} - \mathbf{q}_c) \geq 0$$

$$m \times |\mathbf{q}|$$



Generalized PD Performance

Generalized PD for Rigid Body

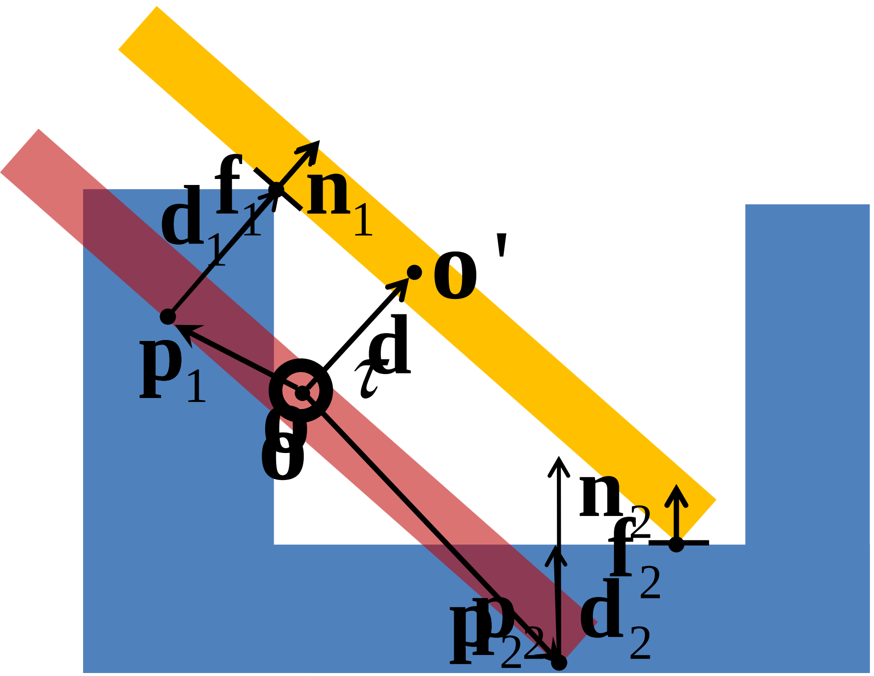
Software Implementations

- Source codes are available
  - http://graphics.ewha.ac.kr/polydepth (translational PD)
  - http://graphics.ewha.ac.kr/hdist (Hausdorff distance and pointwise PD)

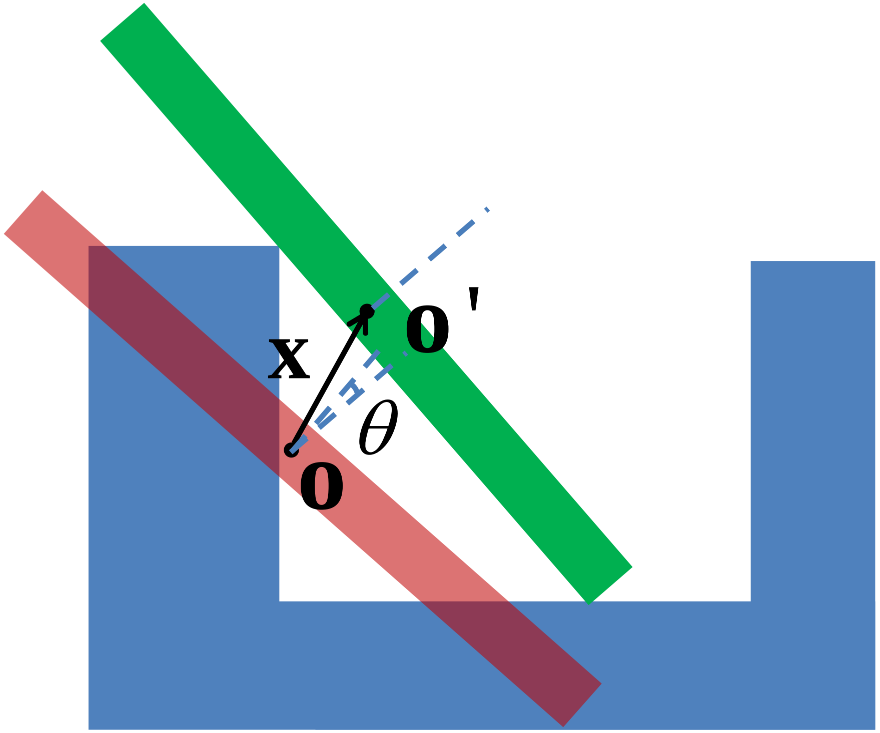
Y. Li, S. Zhang, Y. J. Kim, Six-degree-of-freedom Haptic Rendering using Translational and Generalized Penetration Depth Computation, IEEE WHC 2013

HAPTIC APPLICATIONS

Penalty-based Haptic Rendering using Translational and Generalized PD

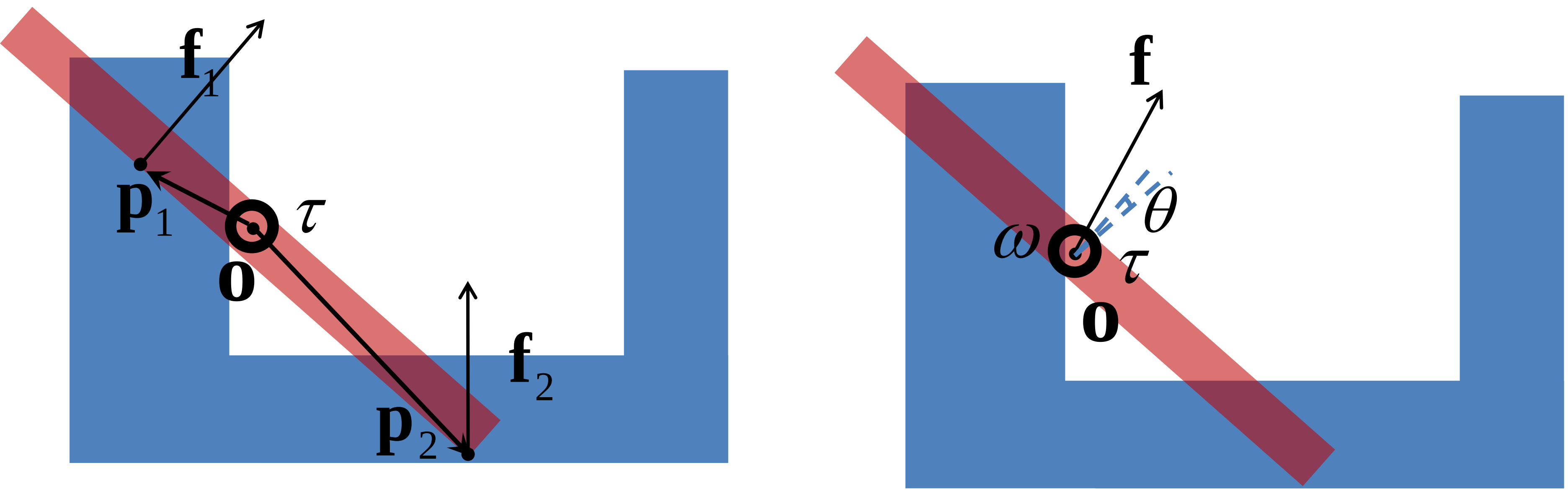


Translational PD



Generalized PD

Penalty-based Haptic Rendering using Translational and Generalized PD



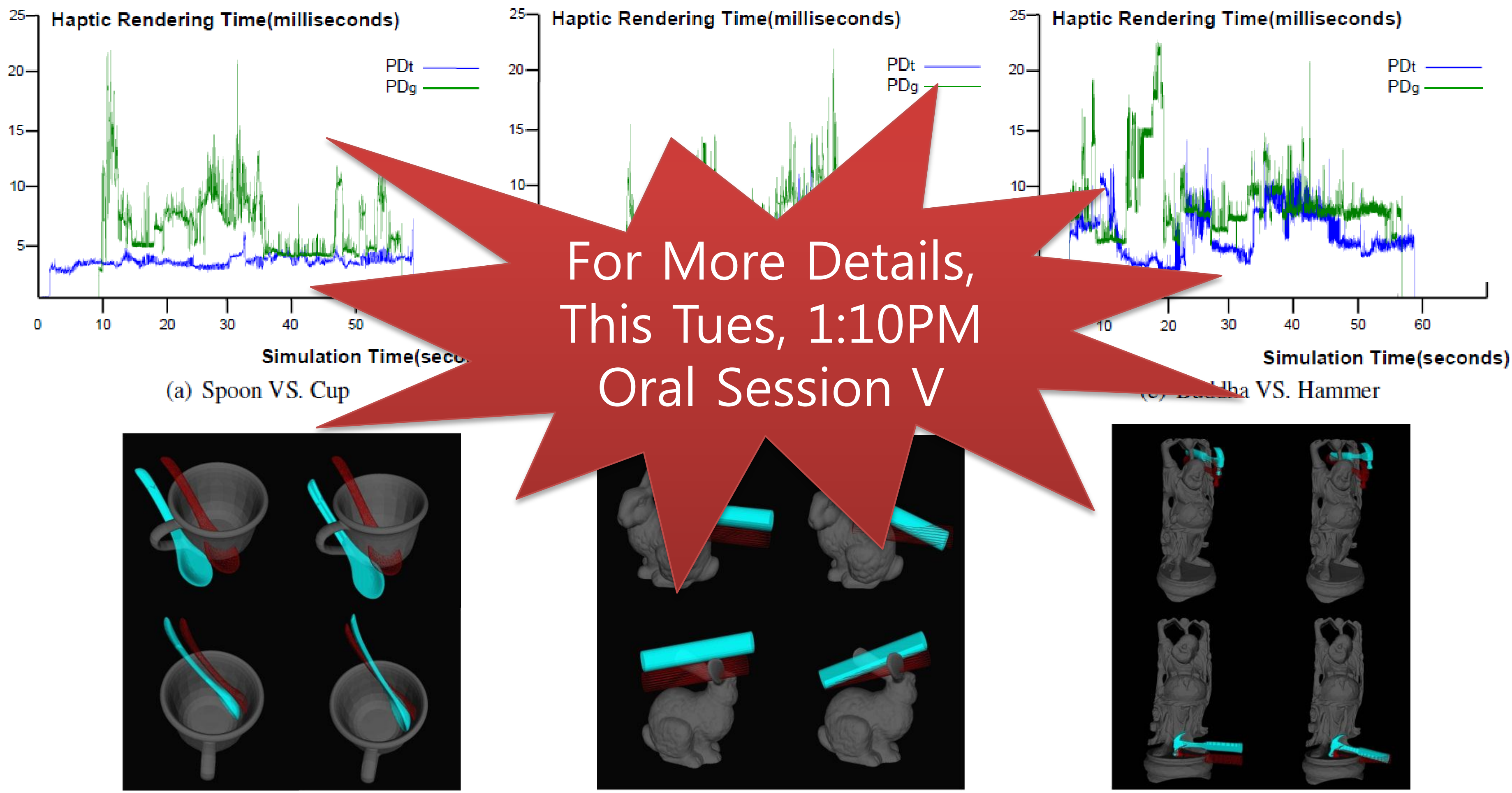
Translational PD

Generalized PD

Benchmarks Setup

6DoF PHANToM Premium 1.5

Performance



For More Details,
This Tues, 1:10PM
Oral Session V

Summary

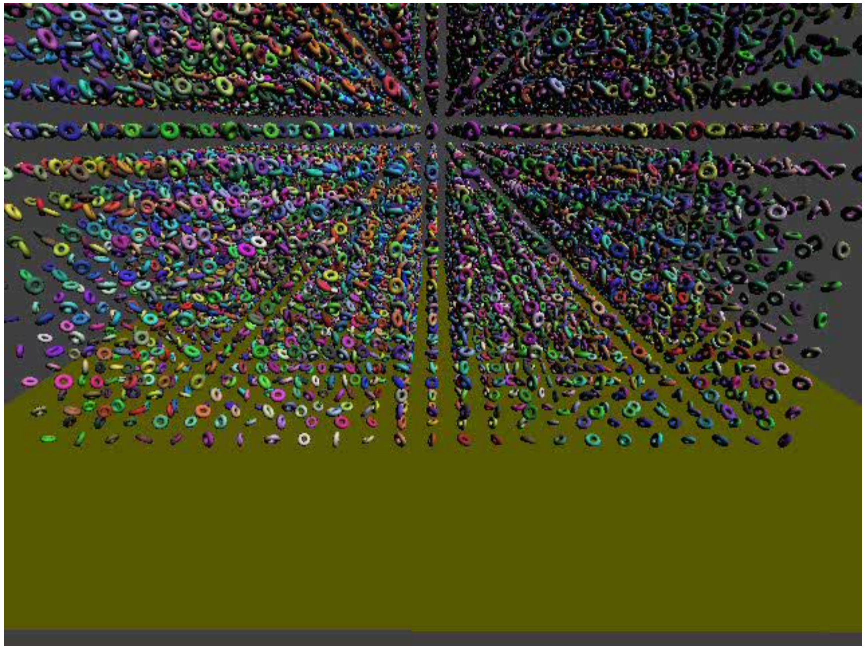
- Pointwise PD
- Translational PD
- Generalized PD

- 6DoF haptic rendering with translational and generalized PD

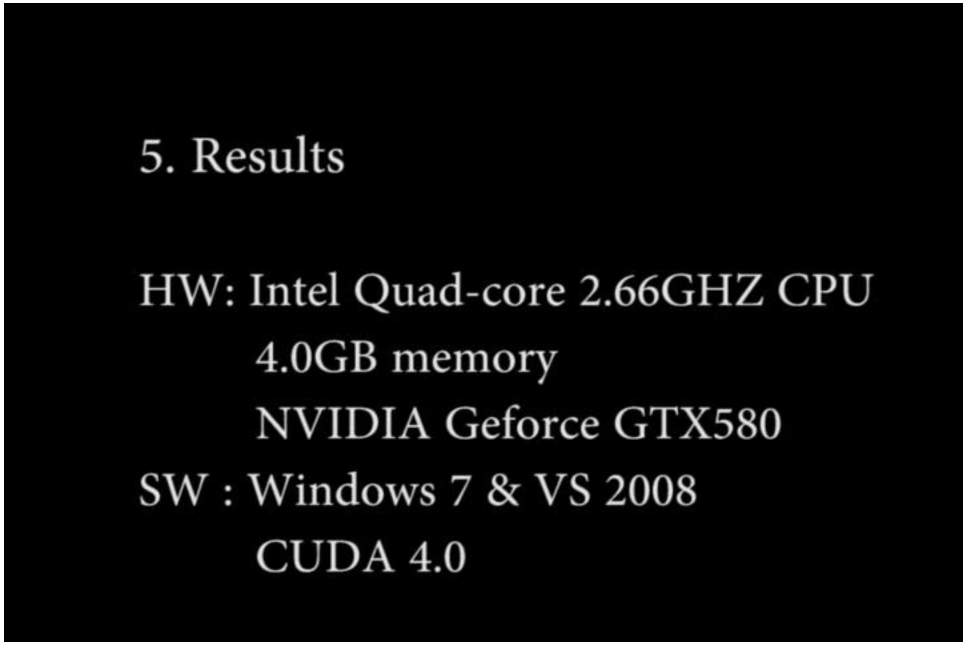
Future Work

- Parallel haptic rendering
  - Asynchronous contact handling
  - GPU-based parallelization
- Haptic rendering for
  - Articulated models
  - Massive models

High Performance GPU-based Collision Queries



Real-time Collision Culling of a Million Bodies on GPUs
ACM Trans on Graphics 2010



Real-time Adaptive Signed Distance Fields for Rigid and Deformable Models on GPUs

Acknowledgements

- Min Tang, Xinyu Zhang, Minkyoung Lee, Yi Li (Ewha)
- Fuchang Liu (NTU)
- Changsoo Je (Sogang)

- KEIT/MKE (IT core research)
- NRF

Thank you for listening!

http://graphics.ewha.ac.kr